GCE Maths Unit 4 – 4.8 Kinematics

Terminology and modelling

Projectile
A particle moving in a vertical plane only under the influence of gravity.

Angle of projection
The angle between the horizontal line and the line on which the particle is projected.

Range
The horizontal distance travelled by the particle.

Time of flight
Time taken for the projectile to reach its horizontal end point.

Air resistance
When an object moves through the air it experiences a resistance due to friction. This is considered as negligible for modelling purposes.

Acceleration due to gravity, $g = 9 \cdot 8$ ms^{-2}
Assumed to be constant. In vector notation, $\boldsymbol{a} = -g\mathbf{j}$, where the unit vector $\mathbf{j}$ is perpendicular to the surface of the earth.

Equations for motion for constant acceleration

Linear constant acceleration		Vector constant acceleration	
$v = u + at$		$\mathbf{v} = \mathbf{u} + \mathbf{a}t$	
$s = ut + \frac{1}{2}at^2$		$\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 \quad (+\mathbf{r}_0)$	
$s = \frac{1}{2}(u + v)t$		$\mathbf{r} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t \quad (+\mathbf{r}_0)$	
$v^2 = u^2 + 2as$		*No vector equivalent*	
s = displacement u= initial velocity v= final velocity	a = acceleration t = time	r_0= initial position r = position vector u = initial velocity	v = final velocity a = acceleration vector t = time

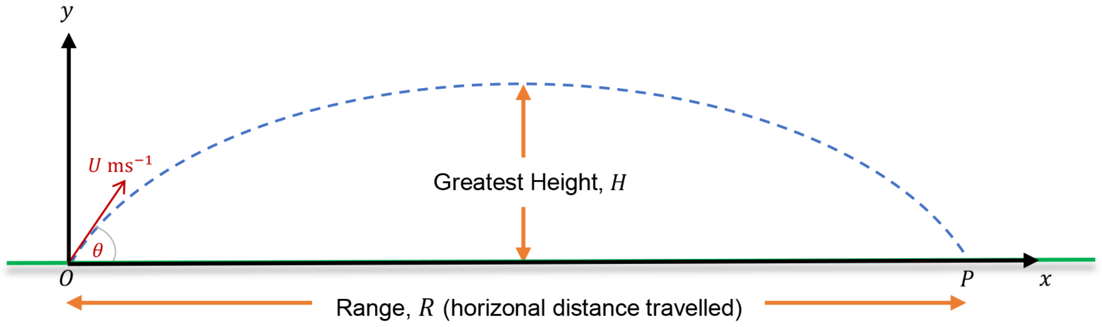
A 2D vector in the form $\boldsymbol{r} = x\mathbf{i} + y\mathbf{j}$ may alternatively be written using a column vector as $\boldsymbol{r} = \binom{x}{y}$.

Projectiles: Equations to be derived

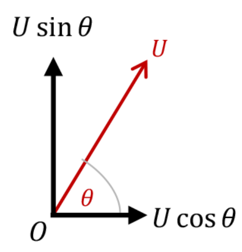
A particle projected from O with speed U ms^{-1} at an angle θ above the horizontal:

- Range, $R = \frac{U^2}{g}\sin 2\theta$
- Time of flight, $T = \frac{2U}{g}\sin\theta$
- Greatest height, $H = \frac{U^2}{2g}\sin^2\theta$
- Equation of path (trajectory) at any point (x,y),

$$y = x\tan\theta - gx^2\frac{(1 + \tan^2\theta)}{2U^2}$$



Resolving

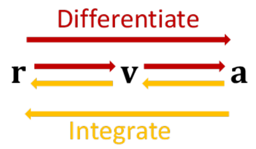


Useful facts

- Horizontally, acceleration is zero, so velocity is constant and therefore $s = ut$.
- At the greatest height, the vertical component of velocity is zero and, therefore, speed is a minimum.
- The path of the projectile is symmetrical.

Variable acceleration using vectors/calculus

$\boldsymbol{r}$, $\boldsymbol{v}$ and $\boldsymbol{a}$ are 2D vectors given in terms of t.



$\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\quad$ $\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$ $\quad$ $\mathbf{r} = \int \mathbf{v}\,dt$ $\quad$ $\mathbf{v} = \int \mathbf{a}\,dt$

Position vector, $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$	Velocity, $\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j}$	Acceleration, $\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j}$
Distance from origin $= \sqrt{x^2 + y^2}$	Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$	Magnitude of acceleration $= \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2}$